Vector Identity (2)

$$\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) = \mathbf{B}(\mathbf{A} \cdot \mathbf{C}) - \mathbf{C}(\mathbf{A} \cdot \mathbf{B})$$

Proof

$$\begin{aligned}
\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) &= \left(\sum_{i=1}^{3} \boldsymbol{\delta}_i A_i\right) \times \left[\left(\sum_{j=1}^{3} \boldsymbol{\delta}_j B_j\right) \times \left(\sum_{k=1}^{3} \boldsymbol{\delta}_k C_k\right)\right] \\
&= \left(\sum_{i=1}^{3} \boldsymbol{\delta}_i A_i\right) \times \left[\sum_{j=1}^{3}\sum_{k=1}^{3} (\boldsymbol{\delta}_j \times \boldsymbol{\delta}_k) B_j C_k\right] \\
&= \left(\sum_{i=1}^{3} \boldsymbol{\delta}_i A_i\right) \times \left(\sum_{j=1}^{3}\sum_{k=1}^{3}\sum_{l=1}^{3} \boldsymbol{\delta}_l \varepsilon_{ljk} B_j C_k\right) \\
&= \sum_{i=1}^{3}\sum_{j=1}^{3}\sum_{k=1}^{3}\sum_{l=1}^{3} (\boldsymbol{\delta}_i \times \boldsymbol{\delta}_l) \varepsilon_{ljk} A_i B_j C_k \\
&= \sum_{i=1}^{3}\sum_{j=1}^{3}\sum_{k=1}^{3}\sum_{l=1}^{3}\sum_{m=1}^{3} \boldsymbol{\delta}_m \varepsilon_{ilm} \varepsilon_{ljk} A_i B_j C_k \\
&= \sum_{i=1}^{3}\sum_{j=1}^{3}\sum_{k=1}^{3}\sum_{l=1}^{3}\sum_{m=1}^{3} \boldsymbol{\delta}_m \varepsilon_{lmi} \varepsilon_{ljk} A_i B_j C_k \\
&= \sum_{i=1}^{3}\sum_{j=1}^{3}\sum_{k=1}^{3}\sum_{m=1}^{3} \boldsymbol{\delta}_m (\delta_{mj}\delta_{ik} - \delta_{mk}\delta_{ij}) A_i B_j C_k \\
&= \sum_{i=1}^{3}\sum_{j=1}^{3}\sum_{k=1}^{3}\sum_{m=1}^{3} (\boldsymbol{\delta}_m \delta_{mj}\delta_{ik} A_i B_j C_k - \boldsymbol{\delta}_m \delta_{mk}\delta_{ij} A_i B_j C_k) \\
&= \sum_{i=1}^{3}\sum_{j=1}^{3}\sum_{k=1}^{3}\sum_{m=1}^{3} \boldsymbol{\delta}_m \delta_{mj}\delta_{ik} A_i B_j C_k - \sum_{i=1}^{3}\sum_{j=1}^{3}\sum_{k=1}^{3}\sum_{m=1}^{3} \boldsymbol{\delta}_m \delta_{mk}\delta_{ij} A_i B_j C_k \\
&= \sum_{i=1}^{3}\sum_{j=1}^{3}\sum_{k=1}^{3} \boldsymbol{\delta}_j \delta_{ik} A_i B_j C_k - \sum_{i=1}^{3}\sum_{j=1}^{3}\sum_{k=1}^{3} \boldsymbol{\delta}_k \delta_{ij} A_i B_j C_k \\
&= \sum_{i=1}^{3}\sum_{j=1}^{3} \boldsymbol{\delta}_j A_i B_j C_i - \sum_{j=1}^{3}\sum_{k=1}^{3} \boldsymbol{\delta}_k A_j B_j C_k \\
&= \sum_{j=1}^{3} \boldsymbol{\delta}_j B_j \left(\sum_{i=1}^{3} A_i C_i\right) - \sum_{k=1}^{3} \boldsymbol{\delta}_k C_k \left(\sum_{j=1}^{3} A_j B_j\right) \\
&= \mathbf{B}(\mathbf{A} \cdot \mathbf{C}) - \mathbf{C}(\mathbf{A} \cdot \mathbf{B})
\end{aligned}$$